

# Analysis of the Filament Labs Castor Three-D Printer Passive Printing Enclosure from a Heat Transfer Perspective

**Aditya Mehrotra**

White Paper Report—Not to be Published

Written in December of 2020

Department of Electrical Engineering and Computer Science

Massachusetts Institute of Technology

Email: [adim@mit.edu](mailto:adim@mit.edu)

**Abstract.** *This white paper looks at the Filament Labs Castor Three-D printer and the addition of a glass and wood enclosure to determine its effectiveness at containing thermal energy. We will compare the difference in power required to maintain the temperature of the heat bed with and without the enclosure theoretically. We will attempt to use empirical measurements to verify the results. We will model the system of the three-D printer as a thermal circuit. At the end of the paper we will discuss our findings and then propose design improvements based on the findings. This paper will also present basic theory on thermal circuits and modeling thermal systems as circuits in the early parts of the paper.*

## I. Introduction

Additive Manufacturing Systems (AMS) have come a long way since they were first introduced. Three-D printers are now able to create parts which can vary widely in strength, flexure, geometry, material, and more. Some of the reasons for the advances in AMS technology include dual extrusion, advanced support algorithms, targeted materials science, and mechanical improvements such as enclosures.

Three-D printer enclosures significantly improve the performance of any AMS. Designed with thermal considerations in mind, the system prevents drafts and temperature changes in the surrounding environment from interfering with the

temperature near the build-surface. The air that surrounds the printer is effectively isolated and captures heat from the printer allowing the filament to cool at a more gradual, uniform rate. The result—due to the major reduction in layer warping and de-lamination that this causes—is a system that can print tougher materials (ABS, Nylon, TPU) and stronger parts.



An enclosure also has the benefit of improving the efficiency of a printer. The insulated surroundings means heating the build-plate requires less energy as less is lost to the surrounding environment which brings us to the focus of this paper.

**I.1.0 Three-D Printer Choice.** This paper will focus on the analysis of the Filament Labs Castor Three-D printer—a printer of partially our own design. Castor is the result of performing systematic and strategic upgrades on an off-the-shelf printer to boost performance, expand capabilities, and increase efficiency. Castor is based on a Tronxy X5SA Pro Three-D printer. Some of the upgrades are listed below and an image of the final system is on the previous page.

- Addition of a glass and wood printing enclosure.
- Replacement of the standard build-plate with a borosilicate glass build-plate.
- The addition of a humidity-controlled filament holder as well as air-tight filament injectors.
- Recalibration of the auto-leveling system and addition of automatic z-offset calibration on startup.

Below is an image of the filament injectors and other upgrades to the X5SA Pro.



**I.2.0 Purpose of Paper.** This paper will focus on the glass enclosure upgrade that was added to the system. We are interested in developing a model that will help us determine the effectiveness of the enclosure we have designed in the following areas.

- Comparison of the efficiency of the printer (power required to maintain bed temperature) before and after the enclosure was added.
- Metrics on the control of the ambient temperature using the build-plate as a source to heat the chamber.

The first sections of this paper will be devoted to a review of the heat transfer equations and modeling techniques we will use in the second half of this paper including a review of heat transfer circuits. The second half of this paper will present the analysis done on the printer itself. We will present our final results and ideas for improving the design of the printer enclosure at the end of the paper. MATLAB code will be included in the appendix so our results and method may be replicated by others. We will finally present avenues for future work.

## II. Overview of Heat Transfer + Thermal Circuits

Much of this section consists of a summary of the information presented in [1] up to the point we start talking about thermal circuits as that formulation is mostly our own. We include this section on an overview of heat transfer to ensure our results can be replicated easily. Note the following important variable definitions.

### Variables:

$\kappa$  = Thermal Conductivity (W/m/°C)  
 $A$  = Cross Sectional Area (m<sup>2</sup>)  
 $t$  = Material Thickness (m)  
 $T$  = Some Representative Temperature (K)  
 $m$  = Mass in kg (kg)  
 $C_p$  = Specific Heat (J/kg/°C)  
 $h$  = Coupling Constant of Conduction (W/m/°C)  
 $\Delta T$  = Temperature Difference (°C)  
 $P_x$  = Power Lost via Method "x" (W)     $Q = \int P_x = \text{Thermal Energy (J)}$

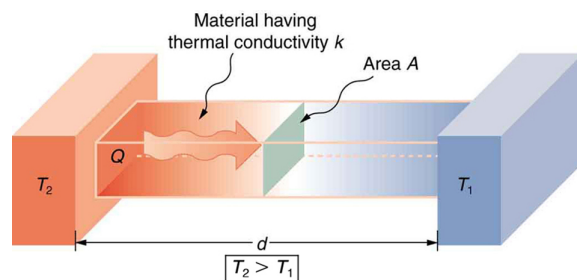
### Constants:

$\sigma = 5.67 \times 10^{-8}$  Radiation Constant (W/m<sup>2</sup>/K<sup>4</sup>)

**II.1.0 A Brief Overview of Heat Transfer.** As presented in [1], heat can only transfer through an environment via three methods—convection, conduction, and radiation. We

will explain these briefly and then provide the equations we will use to model our heat transfer system.

**II.1.1 Conduction as a Mode for Heat Transfer as Described by [1].** Conduction is a mode of heat transfer by which heat is transferred through a material when a temperature gradient is placed across it. Conduction also occurs between two materials that are in perfect contact as heat can transfer from one to the other.



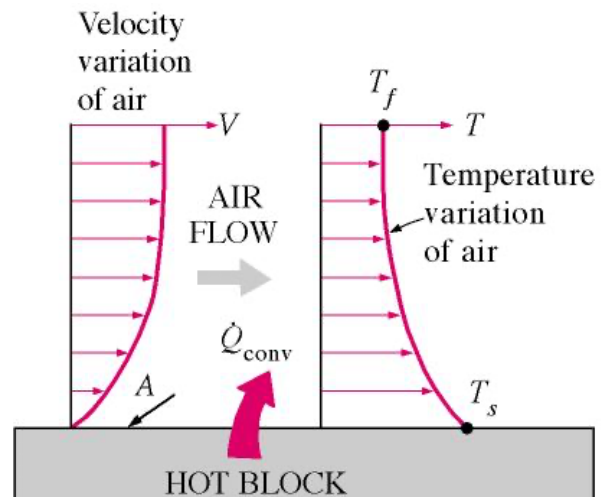
The above diagram from [8] shows the process of thermal conduction. Let's say we have a material with a defined temperature gradient across it. We know that the larger the distance between  $T_1$  and  $T_2$  the more difficult it will be for heat to transfer from one to the other. It also makes sense that a smaller area of contact between the surfaces will lead to less overall energy transfer. We can construct the following relationship between the power transferred and the thermal gradient.

$$P_{cond} = \frac{\kappa A}{t} \Delta T$$

This equation describes the power being transferred from one body to the other. It's important to note that thermal energy always flows from hot surfaces to cold surfaces as the hot particles are higher in energy than the cold particles.

**II.1.2 Convection as a Mode for Heat Transfer as Described by [1].** Convection is the process by which heat is transferred from a solid surface into a moving fluid. As mentioned in I.1.1, conduction is the process by which heat transfers between materials

that are in contact, therefore convection is a special case of conduction with the exception of the moving particles. Now we're probably all familiar with the heat-sink computer-fan combination and know that using a computer fan to create air-flow around the heatsink is more effective than using the heatsink alone. This indicates the velocity of the fluid at the boundary has an impact on convection.



The above diagram from [9] shows convection as we and [1] refer to it. Note that a moving fluid around a solid surface will produce what is called a boundary layer [10]. A boundary layer is a thin layer of fluid close to the surface where the velocity varies from zero at the surface of interaction, to the free-stream fluid velocity at some distance away. In this layer the solid conducts heat to the fluid and the heat is transferred up the layers. Power is constantly being dissipated in this process as the motion of the fluid transfers heat away from the body. Thermal equilibrium is theoretically reached when the thermal power entering the solid equals the thermal power being extracted due to conduction.

$$P_{conv} = h A \Delta T$$

The above equation governs the thermal power transferred due to this process from the solid to the fluid. "h" is a constant that captures all the complicated boundary layer

physics. Typical values for “h” from [1] are listed below.

**Typical “h” Values:**

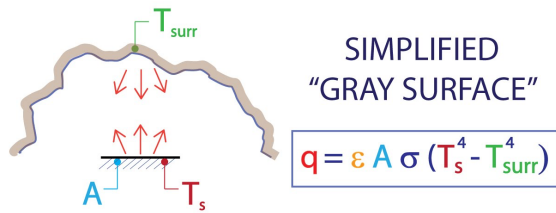
$h \approx 2$  for Indoor Surfaces and “Still” Air  
 $h \approx 5$  for Light Outdoor Breezes  
 $h = 10 \rightarrow 20$  for Windy Conditions

Since our entire system is indoors, we will likely always model “h” as around  $2 \text{ W/m}^2/\text{°C}$ .

### II.1.3 Radiation as a Mode for Heat

**Transfer as Described by [1].** Radiation is the most complicated of all the thermal power transfer methods. Radiation is governed by the emission of EM waves from a body.

#### RADIATIVE HEAT TRANSFER EQUATION



The above diagram from [11] describes this process. The total power lost due to radiation is proportional to the difference between the fourth powers of the temperatures of each of the surfaces. It’s proportional by the exposed area, and two constants. The epsilon constant refers to the emissivity of the surface—how likely is a surface to emit energy. Metals generally have low emissivities because of their tendency to reflect incoming energy. The sigma constant is the Stefan-Boltzmann constant which relates the thermal energy radiated by a black body to its absolute temperature.

According to [1], we can linearize the relationship for small differences in temperature on the absolute scale—our problem fits these constraints. We end up with the following equation.

$$P_{rad} = 4A\sigma T^3 \Delta T$$

“T” is a representative temperature maybe halfway between the hot and cold temperatures. For the example of a house @20°C compared to the outside at 0°C, [1] chose  $T=283\text{K}$ . We will pick our “T” values strategically but this is discussed later.

**II.2.0 Energy Storage in Materials.** We now move on from source [1] to some basic ideas about materials and thermal storage. It’s important to note that in the case of the 3D printer enclosure the inner air increases in temperature as there is a small enough volume where the energy being radiated can heat this air and allow it to *store thermal energy*. Note that the air outside the case is at too high a volume to be effectively considered as storage so we assume that to simply be a perfect source of loss rather than storage.

$$Q = mC_p\Delta T$$

Above is the classic equation for the energy required to heat a mass of material. This will come in handy in a moment when we determine how to model the “storage” of energy component of this system.

**II.3.0 Thermal Circuits.** Thermal circuits allow for more intuitive solving and interpretation of heat transfer problems in some cases. We will utilize thermal circuits as they describe the system of a three-D printer relatively well.

**II.3.1 Power Sources + Resistance.** In any simple circuit we define an “across” and a “through” value for each component. In an electrical circuit this is voltage and current respectively. In terms of a thermal circuit we will define these as the temperature gradient and the power transferred. Any circuit obeys Ohm’s Law. Using that logic we can write convection, conduction, and the radiation in terms of “resistances” to thermal energy flow. Below is radiation.

$$\begin{aligned} P_{rad} &= 4A\sigma T^3 \Delta T \\ \Delta T &= \frac{1}{4A\sigma T^3} P_{rad} \\ V &= R I \\ R_{rad} &= \frac{1}{4A\sigma T^3} \end{aligned}$$



Similarly we can write the following equations for the other modes of heat transfer.

$$R_{rad} = \frac{1}{4A \sigma T^3}$$

$$R_{cond} = \frac{t}{A \kappa}$$

$$R_{conv} = \frac{1}{h A}$$

**II.3.2 Storage Devices.** We can also describe storage of thermal energy as a component of the circuit. That fact is one of the reasons this paper chose to model this system as a circuit. Note the following for a capacitor, charge, and current.

$$q = C V$$

$$\int i \, dt = q$$

We also know that we can have an equation that describes energy required to heat a material.

$$Q = m C_p \Delta T$$

And finally, we know that the following relation between energy and power holds.

$$Q = \int P_x \, dt = \text{Thermal Energy (J)}$$

Therefore, we can describe the air in the casing of the printer as a capacitor where the thermal capacitance is described as the following.

$$C_t = m C_p$$

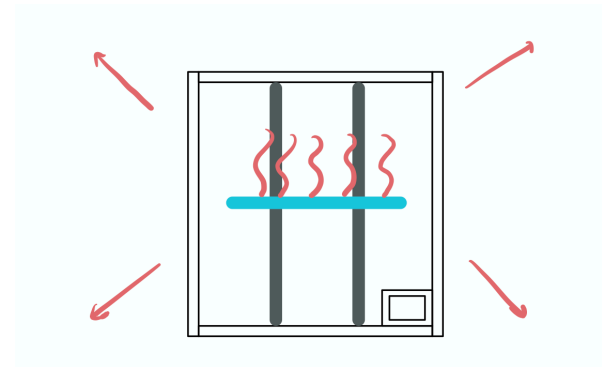
We now have a circuit analog for radiation, conduction, convection, and thermal storage devices.

**II.4.0 Equilibrium Ideas.** The last topic before getting to the actual modeling, assumptions, and calculations is that of equilibrium. The essential concept of this paper is to calculate the thermal power exiting the printer system both with and

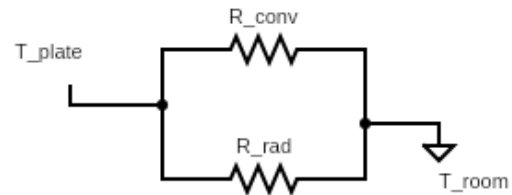
without the enclosure. We can further stipulate that for the printer to maintain a constant bed temperature, the thermal power leaving the system must equal the electrical power entering the system once the system is at equilibrium. We can use this stipulation to compare the efficiency of the printer with and without the enclosure and will do so later in this paper. This idea comes from fundamental conservation of energy laws.

### III. Printer Description + Model

In this section we will describe the structure of the printer with and without the enclosure and our modeling assumptions.



**III.1.0 Printer w/ no Enclosure.** In the case of a printer with no enclosure, we will model the printer as simply a heated build-plate which transmits heat to the surrounding environment which is at room temperature. The only two processes that are relevant here are conduction and radiation. Since the build-plate is directly interfacing with the surrounding environment we can create a very simple circuit to model this system.



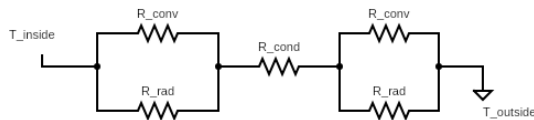
Note that the convection and radiation resistances are in parallel which makes sense as both are contributing to heat loss

and have the same temperature gradient across them for a given surface.

In this case, thermal power is very simply lost to the environment and we will assume that the environment is large and no thermal power is really “stored” by the air in the room (i.e. the thermal capacitance of the room is infinite). Any power transmitted to the environment is simply lost.

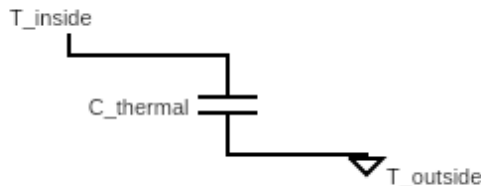
**III.1.1 Printer + Enclosure.** To add the effects of the printer enclosure we need to consider what occurs at each of the walls and to the air within the box itself.

The enclosure’s walls will draw thermal energy from the air within the casing. Then they will conduct this thermal energy to their outward-facing surfaces where the energy will be discharged into the environment. For each wall, we can model its thermal interactions with the below circuit.



The temperatures  $T_{\text{inside}}$  and  $T_{\text{outside}}$  refer to the ambient temperatures inside and outside the enclosure respectively.

Lastly, we can also model the air inside the case as a capacitor with a thermal capacitance “ $C_{\text{thermal}}$ .”



**III.2.0 General Modeling Assumptions.** We will make the following assumptions when performing our calculations and while interpreting the results of these calculations.

- The room volume is large compared to the printer volume so the thermal capacitance of the room is infinite.
- The enclosure is completely air-tight and there are no seams between the materials or corners.
- The temperature differences are small compared to the absolute scale and linearization can be employed for radiation.
- The electric power required to keep the build-plate at the correct temperature is equal to the thermal power lost to the environment at steady-state.
- Assume  $h = 2 \text{ W/m}^2/\text{°C}$  for all the following calculations.
- The temperature of the build plate is being maintained at a constant, desired value.
- Due to the presence of fiberglass insulation ( $k=0.043$ ), the build-plate only loses thermal energy from the upper surface.
- The entire thermal system is at steady-state.
- The build-plate is centered on the z-axis (vertically) and equidistant from all the walls.

We will discuss the validity of these assumptions in the analysis section and methods to improve our modeling. We will also note the following information about the enclosure, summarized below.

Material	Qty.	$t$ (m)	$A$ ( $\text{m}^2$ )	$K$ ( $\text{W/m}^2\text{°C}$ )
Glass	(4x)	0.00238125	0.464515	0.8
Wood	(2x)	0.01905	0.371612	0.12

Also note that the build-plate is 330x330mm borosilicate glass. A summary of other values in SI units is located in the MATLAB script in Appendix B.

## IV. Calculations

This section presents both the mathematical analysis of the stand-alone printer as well as that of the printer with the enclosure. See Appendix A for full schematics of the equivalent thermal circuits for both systems.

This section will take the approach of explaining the MATLAB code and how we arrived at both the inputs and outputs.

```
% Environment Setup
%_____BE SURE TO RUN THIS SECTION BEFORE ANY CALCULATION_____
clear all
close all
clc

% Define Some Initial Constants
%THERMAL CONSTANTS
sigma = 5.67*10^(-8); %W/m^2/K^4
Cp_air = 1005; %J/kg.K
d_air = 1.225; %kg/m^3 assume sea-level
k_wood = 0.12; %W/m^2/°C
k_glass = 0.8; %W/m^2/°C
%PROBLEM-SPECIFIC CONSTANTS
h = 2; %W/m^2/°C
A_plate = 0.1089; %m^2
A_glass = 0.464515; %m^2
A_wood = 0.371612; %m^2
t_glass = 0.00238125; %m
t_wood = 0.01905; %m
v_air = 0.28316847; %m^3 assume it fills case
m_air = d_air*v_air; %kg

%BOUNDARY CONDITIONS
T_plate = 70; %°C
T_room = 22; %°C, or 70°F
```

**IV.1.0 Environment Setup.** We first define constants that are relevant to the system. Starting with constants that are not system-specific, that includes sigma and the material properties of the glass, wood, and air.\*

*\*Notice the units for these constants, Cp\_air is in J/(kg.K) because sigma and other constants are given in W.*

We now want to write the system-specific constants' values so the rest of the code can be written in variable names. As discussed previously, we will use  $h = 2 \text{ W/m}^2/\text{°C}$  as the coefficient of thermal convection for this problem as we are indoors and there should be relatively little air flow. The areas and thicknesses of the glass panels, wood panels, and the build-plate are all defined here as well. Finally, we calculate the volume and mass of the air in the enclosure.\*

*\*It's also important to note the units here, every dimension must be given in meters,  $\text{m}^2$ , or  $\text{m}^3$  for consistency. These were converted from their original values in inches.*

Finally we place the boundary conditions of the problem at the bottom. This includes the temperature of the plate and the air outside the enclosure. This script will use the circuit model to solve for the following.

- Power dissipation at steady-state of the three-D printer alone.

- Power dissipation at steady-state of the three-D printer w/ enclosure.
- Power savings that the enclosure provides.
- The temperature of the air inside the enclosure.
- The difference in temperature between air inside and outside the enclosure.
- The transient analysis of time it takes the enclosure to heat up to its steady-state temperature.

```
% CALCULATION 1: Printer w/ no Enclosure

%first calculate T_REP
T_REP = (T_plate+T_room)/2+273.15; %MUST BE IN KELVIN

%calculate R_Rad + R_Conv
R_rad_plate = 1/(4*A_plate*T_REP^3*sigma);
R_conv_plate = 1/(h*A_plate);

%calculate equivalent resistance
R_TOTAL_NO_ENCLOSURE = R_rad_plate+R_conv_plate/(R_conv_plate+R_rad_plate);

%calculate the power loss in W
P_LOST_NO_ENCLOSURE = (T_plate-T_room)/R_TOTAL_NO_ENCLOSURE
```

$$R_{rad} = \frac{1}{4A\sigma T^3}$$

$$R_{cond} = \frac{t}{A\kappa}$$

$$R_{conv} = \frac{1}{hA}$$

**IV.2.0 Stand-Alone Printer.** As discussed earlier, the stand-alone printer loses thermal energy due to both convection and radiation. We will describe radiation first.

To determine the thermal resistance between the glass build-plate and the air due to radiation we first need to calculate a representative "T\_REP" value to use in the formulas reproduced above. We will set this "T\_REP" for all parts of the problem to be the Kelvin temperature halfway between the build plate temperature and the outside environment temperature. This is the constant value we will linearize around for the rest of this paper.\*

*\*Note that we could be more accurate in picking this T\_REP. For example, we could pick multiple T\_REPs, one for the interaction between the plate and the air inside the case, one for the interaction between the air inside the case and the enclosure, and one for the interaction between the enclosure and the air outside the case. We will not do this for simplicity.*

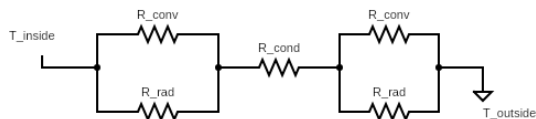
After this, calculating the thermal resistances is a matter of plugging in the values and accounting for convection. We can then combine these resistances in parallel and put

a voltage of  $(T_{\text{plate}} - T_{\text{room}})$  across the combination and solve the very simple circuit. See Appendix A for the thermal circuit diagram.

**IV.3.0 Printer w/ Enclosure.** For the printer with the enclosure we will break up the explanation into a few parts as the code becomes a little more complex here.

```
%__CALCULATE THERMAL RESISTANCE OF GLASS__
%NOTE: We will change T_REP later if we can
R_rad_inner_glass = 1/(4*A_glass*T_REP^3*sigma);
R_conv_inner_glass = 1/(h*A_glass);
R_rad_outer_glass = 1/(4*A_glass*T_REP^3*sigma);
R_conv_outer_glass = 1/(h*A_glass);
R_cond_glass = t_glass/(A_glass*k_glass);

%combine and simplify resistance for glass
R_INNER_GLASS = R_rad_inner_glass*R_conv_inner_glass/(R_rad_inner_glass+R_conv_inner_glass);
R_OUTER_GLASS = R_rad_outer_glass*R_conv_outer_glass/(R_conv_outer_glass+R_rad_outer_glass);
%GET R_GLASS NET
R_GLASS = R_INNER_GLASS+R_OUTER_GLASS+R_cond_glass;
```



**IV.3.1 Thermal Resistance of Glass.** We will solve the more complicated problem by first calculating the total thermal resistance of each of the walls accounting for all modes of heat transfer. That is, we will calculate a single resistance value for glass and wood by combining the network above into a single equivalent resistor.

To do so we must determine the convection and radiation interactions inside and outside the surface for glass, and the conduction through the glass. Since we already have calculated a representative temperature  $T_{\text{REP}}$ , this simply requires plugging in the constants we defined earlier, the area and thickness of the glass, and the  $k$ -value.

We then combine the inner thermal resistance due to radiation and thermal resistance due to convection in parallel. Then, we do the same for the outer and add the three resistances in series to get a final thermal resistance of the glass.

**IV.3.2 Thermal Resistance of Wood.** Note that the calculation of  $R_{\text{WOOD}}$  is done in exactly the same way as it was done for  $R_{\text{GLASS}}$ . The only things that change are the thickness, area, and thermal conductivity.

```
%__CALCULATE THERMAL RESISTANCE OF WOOD__
%NOTE: We will change T_REP later if we can
R_rad_inner_wood = 1/(4*A_wood*T_REP^3*sigma);
R_conv_inner_wood = 1/(h*A_wood);
R_rad_outer_wood = 1/(4*A_wood*T_REP^3*sigma);
R_conv_outer_wood = 1/(h*A_wood);
R_cond_wood = t_wood/(A_wood*k_wood);

%combine and simplify resistance for wood
R_INNER_WOOD = R_rad_inner_wood*R_conv_inner_wood/(R_rad_inner_wood+R_conv_inner_wood);
R_OUTER_WOOD = R_rad_outer_wood*R_conv_outer_wood/(R_conv_outer_wood+R_rad_outer_wood);
%GET R_WOOD NET
R_WOOD = R_INNER_WOOD+R_OUTER_WOOD+R_cond_wood;
```

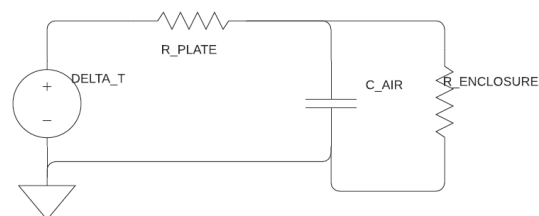
We now have an equivalent thermal resistance for both wood and glass that captures all the modes of heat-transfer.

**IV.3.3 Building the Circuit.** The next step is to now take these individual resistances and fully model the enclosure. The enclosure has  $(4x)$  glass walls, and  $(2x)$  wood walls.

```
%__CALCULATE THERMAL RESISTANCE OF (4x) GLASS & (2x) WOOD__
R_TOTAL_ENCLOSURE = 1/(2/R_WOOD+4/R_GLASS)+R_TOTAL_NO_ENCLOSURE;

%CALCULATE TOTAL POWER LOSS @ STEADY STATE
P_LOST_ENCLOSURE = (T_plate-T_room)/R_TOTAL_ENCLOSURE
```

Essentially, we want to put all of the resistances for the walls in parallel as they all have the same temperature differential across them and all are paths through which heat can transmit from the interior to the exterior of the enclosure.



When we combine those resistances in parallel we get the above circuit. At steady-state, no current should flow through the capacitor so  $R_{\text{PLATE}}$  and  $R_{\text{ENCLOSURE}}$  will be in series at that point. We calculated  $R_{\text{PLATE}}$  in IV.2.0 and we just calculated  $R_{\text{ENCLOSURE}}$ . We add these resistances together and divide the temperature difference by these resistances to get the steady-state thermal power exiting the system.

We can finally calculate the temperature inside the enclosure by determining the voltage difference across the capacitor at steady state and adding that to the outside



```
%CALCULATE TEMP OF AIR INSIDE
T_INSIDE = T_plate-P_LOST_ENCLOSURE/R_TOTAL_NO_ENCLOSURE

%CALC THERMAL CAPACITANCE OF AIR
C_AIR = m_air*Cp_air;
%ENERGY STORED IN THE AIR
ENERGY_STORED_AIR = C_AIR*(T_INSIDE-T_room); %assuming air starts @ room temp
```

temperature. Similarly, we could also take the plate temperature and subtract the temperature drop across the R\_PLATE resistor to get the temperature of the air inside the case.

**IV.4.0 Transient Analysis.** Note that we can take this circuit and create a transfer function for the voltage across the capacitor compared to the voltage input. We can then step this transfer function to get the transient heating of the air inside the enclosure.

$$C_t = m C_p$$

First, we can calculate the thermal capacitance of the air in the enclosure using the above formula reproduced from II.3.2. And then we create and step the transfer function. Note “s” is the Laplace transform variable.

$$H(s) = \frac{\frac{1}{sC_t} || R_{enclosure}}{\frac{1}{sC_t} || R_{enclosure} + R_{plate}}$$

```
% TRANSIENT AND OTHER VALUES

%calculate power savings w/ enclosure
P_SAVED = P_LOST_ENCLOSURE-P_LOST_NO_ENCLOSURE
%temp diff inside and outside enclosure
T_DIFF = T_INSIDE-T_room

%setup the system transfer function
s = tf('s');
H_OUT = 1/(s*C_AIR)*1/(2/R_WOOD+4/R_GLASS)/(1/(2/R_WOOD+4/R_GLASS)+1/(s*C_AIR));
H_SYS = H_OUT/(R_TOTAL_NO_ENCLOSURE+H_OUT);

%plot graph that describes temperature of the air as a function of time
opt = stepDataOptions('StepAmplitude', (T_plate-T_room));
figure;
hold on;
step(H_SYS,opt);
xlabel('Time in (s)');
ylabel('Temp Increase of Air above Room Temp (°C)');
title('Temperature of Air in Enclosure vs Time');
hold off
```

## V. Analysis + Interpretation of Results

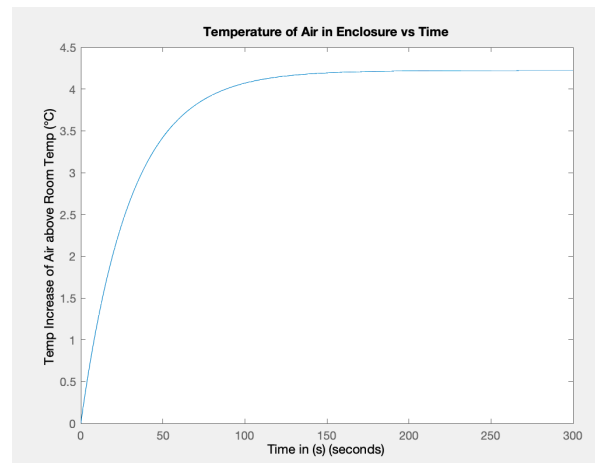
The full commented code file for the above calculations is located in Appendix B. This section will present the outputs of the code-file and interpret their results. We will be careful to account for the assumptions made in III.2.0 when interpreting.

**V.1.0 MATLAB Code Outputs.** Note that the outputs of the MATLAB code are summarized in the table below for reference.

Output	Value	Unit
P_LOST_NO_ENCLOSURE	48.9931	W
P_LOST_ENCLOSURE	44.6842	W
T_INSIDE	26.2215	°C
P_SAVED	-4.3088	W
T_DIFF	4.2215	°C

\* Note that at steady state, all of the power calculated in P\_LOST\_NO\_ENCLOSURE and P\_LOST\_ENCLOSURE is lost to the environment outside the printer.

Our calculation seems to predict that the presence of the enclosure reduces the power lost to the environment outside the printer from the build-plate by about 4.31W. It also seems to predict the temperature inside the enclosure when the build-plate is at 70°C and the environment is at 22°C will be close to 26.22°C.



The above graph is the transient analysis of the system and it seems to predict if the build-plate is held at a constant 70°C and the environment is at 22°C, the system will reach thermal equilibrium in about 160s or a little under 3min.

We now want to experimentally measure key factors that will allow us to verify the above model or generate a new one.

## VI. Experimental Design

Let's start by reminding ourselves of the factors we are actually interested in which include the following as listed in I.1.0.

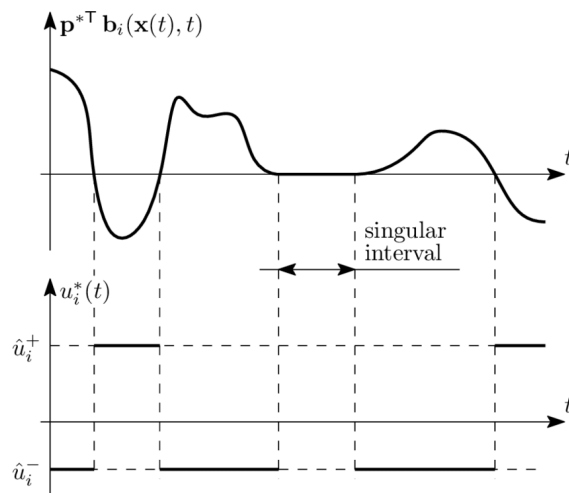
- Comparison of the efficiency of the printer (power required to maintain bed temperature) before and after the enclosure was added.
- Metrics on the control of the ambient temperature using the build-plate as a source to heat the chamber.

**VI.1.0 Procedure + Measurements.** To compare the system's performance with the presence of the enclosure to its performance without, we devised the following simple experiment.

The control group will be the Tronxy X5SA printer on its own and it will be compared to the experimental group which is the X5SA in the enclosure as seen in Section I. For both groups, we will do the following.

- Set the build-plate preheat to 70°C.
- Wait 1hr 15min for temperatures to stabilize.

Note that the printer operates on a “bang-bang” control model which means, at steady-state, it provides full power to the build plate for some number of seconds during a set period of time. The concept is displayed in the following image from [14].



We will make the following measurements at steady-state.

- Measure the  $T_{ON}$  and  $T_{OFF}$  times.
- The voltage and current being supplied to the build-plate.
- The temperature at the top of the borosilicate glass print-bed.
- The temperature at the top of the aluminum heat-bed.
- The temperature at the bottom of the aluminum heat-bed.
- The humidity and temperature of the outside environment, and the temperature within the enclosure.

We will use these metrics to calculate the steady-state-power being used to keep the build-plate at 70°C.

**VI.1.1 Measuring Equipment + Procedure.** We will use the following pieces of test equipment and procedures for their utilization.

*VI.1.1.(a) No-Contact Current Meter.* We will use a no-contact TRMS current meter such as the one below to measure the current going through the wires to the build-plate [15].



The above multimeter has a feature where the screen turns red while measuring a non-zero current and shuts off when no current is being sensed. We will use the screen color change as the indication of when to stop and start the timer at steady-state. For example, for the  $T_{ON}$  time measurement we will start the timer when the screen backlight goes “on,” and stop the timer when it goes “off.” We will also use this multimeter to measure

the voltage at the terminals of the build-plate.

*VI.1.1.(b) Wireless Digital Thermometer.* We will use an outdoor/indoor dual ambient-temperature thermometer to make environmental temperature measurements [16].



The above system uses one unit to measure the temperature in the environment where the display is located, and the wireless unit measures the temperature in another environment. We will place the wireless unit inside the glass-case of our printer and monitor the temperature remotely.

*VI.1.1.(c) Hygrometer.* We will use the hygrometer in [17] to measure the humidity in the surrounding environment.

*VI.1.1.(d) Infrared Thermometer.* We will use an infrared thermometer like the one in [18] to measure the temperature of the build-plate, and heat-bed.

**VI.2.0 Pre-Experiment Notes.** The following are important points to consider.

- We will conduct the experiment in a dark room so that sunlight and the greenhouse effect may not interfere with the temperature inside the enclosure.
- We will place the build-plate in the center of its z-axis to heat the enclosure evenly.
- We will place the thermometer in one corner of the enclosure to ensure accurate temperature readings.

It's also important to note that this experiment only works and is only useful because the printer has an in-built feedback control system attempting to maintain the

temperature of the build-plate at a constant 70°C.

**VI.3.0 Data + Results.** The full data are listed in Appendix C and in [19]. This section will point out key information obtained from the study and will then compare this data to our mathematical predictions.

**VI.3.1 No-Enclosure Results.** The following data comes from the X5SA without the enclosure.

DATA:	CONTROL	Value	Units	Precision	Conversion
CURRENT:	T_ON	5.77	(s)	(+/- 0.005s)	
	T_OFF	4.71	(s)	(+/- 0.005s)	
VOLTAGE:	SURFACE_TEM	156.4	(°F)	(+/- 0.05°F)	69.11111111 (degC)
	HEAT_BED_TEM	173.7	(°F)	(+/- 0.05°F)	78.72222222 (degC)
*measured	HEAT_BED_TEM	165	(°F)	(+/- 0.05°F)	73.88888889 (degC)
	TEMP_IN	N/A	(°F)	N/A	#VALUE! (degC)
	TEMP_OUT	66	(°F)	(+/- 0.05°F)	18.88888889 (degC)
	HUMIDITY	25	(%)	(+/- 0.5%)	
	*measured @ 500 Hz *Account for human reaction time in the time trials				

We can now use this to calculate the control duty-cycle as well as the steady-state average power dissipation.

CALCS:	CONTROL	Value	Units	Precision	Conversion
	Duty Cycle	0.55	unitless	(+/- 0.005)	
	Stead-State-Pow	125.6	(W)	(+/- 0.05)	
	*calculated w/ Precision accounted for				
	*BECAUSE OF MEASUREMENT TECHNIQUE, HUMAN REACTION TIME SHOULD HAVE NO NET EFFECT				

We measured an average power being sent to the build-plate without the enclosure at steady-state to be around 126W.

**VI.3.2 w/ Enclosure Results.** The following data comes from the X5SA with the enclosure.

CURRENT:	EXPERIMENTAL Value	Units	Precision	Conversion
	9.34 T_ON	4.66 (s)	(+/- 0.005s)	
	T_OFF	5.94 (s)	(+/- 0.005s)	
VOLTAGE:	SURFACE_TEM	157.1 (°F)	(+/- 0.05°F)	69.5 (degC)
*measured	HEAT_BED_TEM	172.6 (°F)	(+/- 0.05°F)	78.11111111 (degC)
	HEAT_BED_TEM	162 (°F)	(+/- 0.05°F)	72.22222222 (degC)
	TEMP_IN	76 (°F)	N/A	24.44444444 (degC)
	TEMP_OUT	65 (°F)	(+/- 0.05°F)	18.33333333 (degC)
	HUMIDITY	25 (%)	(+/- 0.5%)	
	*measured @ 500 Hz *Account for human reaction time in the time trials			

We can now use this to calculate the control duty-cycle as well as the steady-state average power dissipation.

EXPERIMENTAL Value	Units	Precision	Conversion
Duty Cycle	0.44 unitless	(+/- 0.005)	
Stead-State-Pow	99.5 (W)	(+/- 0.05)	
	*calculated w/ Precision accounted for		
	*BECAUSE OF MEASUREMENT TECHNIQUE, HUMAN REACTION TIME SHOULD HAVE NO NET EFFECT		

We measured an average power being sent to the build-plate with the enclosure at steady-state to be around 100W.

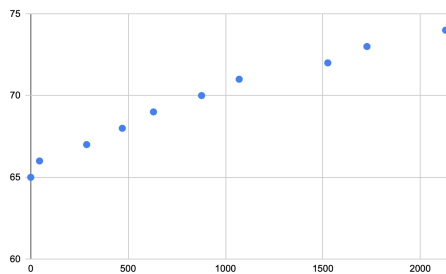
**VI.3.3 Quick Comparison.** We can now calculate the power saved attempting to maintain the 70°C build-plate temperature when using the enclosure as opposed to when not using the enclosure.

FINAL CALCUL. Value	Units	Precision	Conversion
Power Savings	26.1 (W)	(+/- 0.05)	
Temp/Time (cool)	0.00296108291 (°C/s)	(?)	

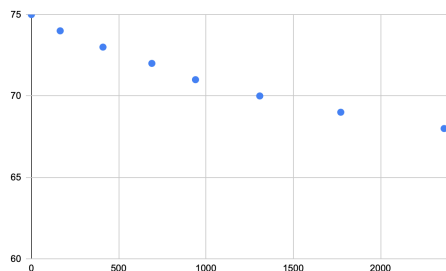
That's a total power savings of close to 26W. That's a significant amount of power savings.

**VI.3.4 Temperature Within the Enclosure.** We were also interested in the temperature of the air within the enclosure compared to the temperature of the surrounding environment. During the trial with the enclosure we measured an exterior temperature of 18.3°C and a build-plate surface temperature of 69.5°C. The MATLAB model from Section IV predicts (with these values) that the temperature of the air inside the enclosure should be 22.7971°C. We measured a final temperature inside the glass case at steady-state of around 24°C.

**VI.3.5 Temperature Transient Within the Enclosure.** We also attempted to measure the heating and cooling transients within the enclosure and produced a graph for heating.



And a graph for cooling (shown below).



These graphs clearly show our transient analysis from V.1.0 is also not accurate.

**VI.3.6 Power Measurements.** The MATLAB also predicted the steady-state power lost to the environment with the enclosure would be around 47W. Our results indicate at steady-state with the enclosure, the power being supplied to the build-plate is around 100W.

The results from the transient measurement, power estimation, as well as VI.3.4 indicate there are issues in our measurement technique, or our model from Section V. There's clearly a discrepancy here that will be discussed in the next section.

## VII. Verification + Analysis

The conflicting results from sections V and VI warranted one final, mini-experiment to ballpark the power usage numbers we are looking for. We took the smart-plug shown in [20] and used it to monitor the power the printer in the enclosure was using at steady-state as well as the power used during the heating phase.

This system is capable of measuring an average power while on, we will use this for the steady-state power measurement for accuracy.

Stage	Value	Unit
No Heating	12	W
Heating Stage Initial	316	W
Heating Stage Middle (@ 61°C)	277	W
Heating @ 70°C	267	W
Steady State (avg.)	120	W

*\*NOTE: According to Belkin, the "avg power" field will take around 24Hrs to update. We left the printer on until the Belkin meter calculated an average power. The data from the WeMo is presented in Appendix D.*

According to the WeMo (Belkin) plug data. The printer used an average power of around 120W while inside the enclosure with the build-plate set to 70°C. This is after leaving



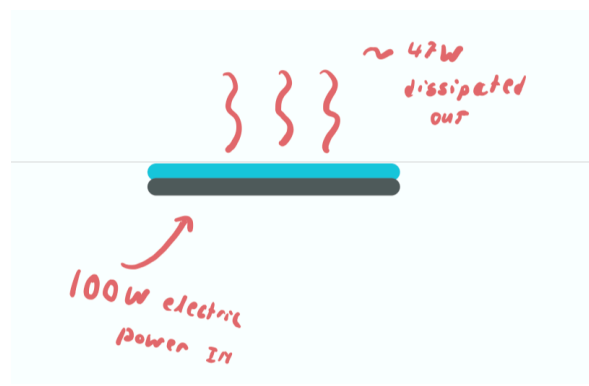
the printer on for about 4Hrs. If we adjust for the 12W we measured before turning the heater on, and we adjust for the fact that the WeMo is reporting the day average which includes the heating cycle. An estimate of 100W for power to the build-plate at steady-state seems plausible.

**VII.1.0 Analysis + Possible Errors in Modeling.** We now attempt to combine the results and calculations above into a final model we might be able to use for our printer. We will specifically do this by attempting to figure out what components we missed out on during modeling and attempt to account for them in this section.

**VII.1.1 Accounting for the Thermal Properties of the Build-Plate.** We will start by determining if the thermal properties of the build-plate itself could be a factor. We initially assumed the following.

- The electric power required to keep the build-plate at the correct temperature is equal to the power the build-plate loses to the environment at steady-state.

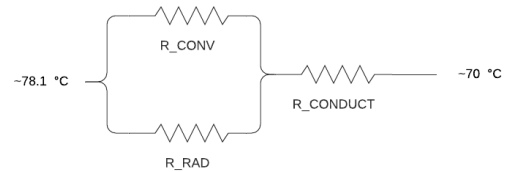
This may technically be true but our model doesn't account for the inner workings of the plate itself.



Now we want to use the data we collected to see if we can account for the discrepancy in power from interactions within the build-plate itself.

Material	k-Value	Thickness	Area (A)
Borosilicate Glass Plate	1.14 W/°C	3 mm	(330mm) <sup>2</sup>
Aluminum Heat-Bed	205 W/°C	~3 mm	(330mm) <sup>2</sup>

Note that we also measured the temperatures on both sides of the borosilicate-glass print-bed. All the following calculations will assume the printer is in the enclosure.



We can now calculate the approximate power loss from conduction and from radiation and convection through this system. Using a T\_REP of 74°C we get around 10W of power dissipated through this system. That still leaves around 43W unaccounted for.

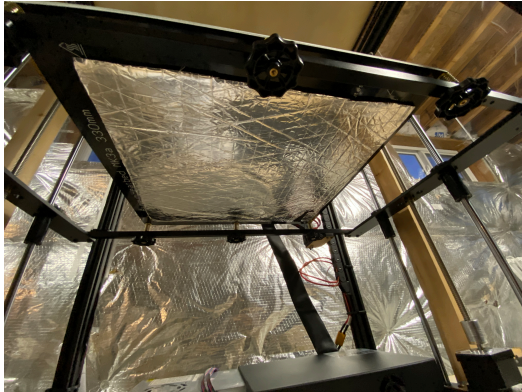
**VII.1.2 Affect of Linearization.** We want to perform a quick comparison between the linearized radiative losses and the value of power dissipated for non-linearized losses.

$$P_{rad} = \epsilon \sigma A (T_1^4 - T_2^4)$$

Above is the formula for the power lost due to radiation without linearization. For our system, we assume the plate is borosilicate-glass where  $\epsilon \sim 0.9$  as with most non-conductors. If we assume  $T_1 = 70^\circ\text{C}$  and  $T_2 = 24.4^\circ\text{C}$  (and we convert to Kelvin) that gives us a power loss to the environment of around 33.4W. On the contrary if we use the linearized formula for radiation we get 35.9W which means linearization is likely not the source of the error here.

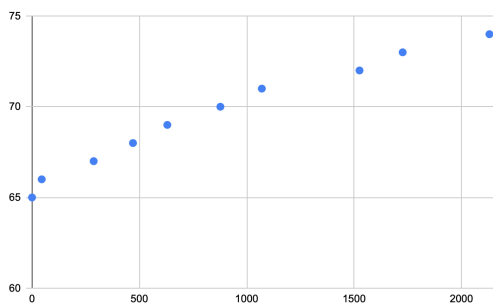
**VII.1.3 Logic + Further Research.** Now let's think about what we know. First, we know that the temperature inside the build-chamber was estimated to be 2°C lower than was measured. We also estimate the power use to be around 43W less than the measured power use. It is possible that we can attribute this difference to the fact that the under-side of the build-plate actually has some exposed area that was not accounted for in the calculations (see below for pictures)

and due to linearization and other small defects in the model.



But we really could've only said this if the power estimated matched the measured power.

The major issue here is the 43W discrepancy. The only pieces of data pointing to an answer comes from the transient analysis. The transient analysis predicted the system would heat the chamber in less than 5 minutes, but when measured we saw it took close to 30 min to actually heat the air in the enclosure to its steady-state temperature.



This could've indicated that the thermal capacitance of the air is much higher than we originally thought it was but that would not explain the difference in steady-state power dissipation. If we add the significant increase in steady-state power dissipation we might conclude that the model for the enclosure could be too insulating. In that the enclosure is leaking a lot more thermal power into the environment than we modeled.

But the other problem is the data from without the enclosure. Our MATLAB prediction says without the enclosure we estimate 51W of power dissipation to the environment while we measured around 126W of electric power being used at steady-state.

**VII.2.0 Overall Analysis.** Based on the experiments, model, and current knowledge on the topic of thermodynamics we can't really speak to what the issue with our model is. Perhaps after taking a thermodynamics class (like MIT's 2.005) we may be able to re-visit this problem and determine in a larger capacity where the errors in the model are. We can summarize some of our initial ideas on the possible issues as the following.

- For the glass enclosure, we should account for panel gaps, drafts, and other areas that could cause more energy to leak from the enclosure.
- We should account for all portions of the exposed area that could cause thermal power dissipation as well as utilize a more accurate model for radiation than the linearized circuit model.
- A better model for the conversion of electrical power to thermal power might be advisable as would a better model of the thermal interactions within the build-plate itself.
- We should re-evaluate if a circuit is the best way to model this system or if it is even an accurate model of this system.
- We should. Further investigate if *our* circuit model captures the details in an accurate manner as it may not even though a circuit might be an "ok" model. We say this specifically in terms of the transient analysis and calculation of thermal capacitance of the air within the enclosure.

To answer the above questions, more extensive knowledge of thermodynamics and the details of heat-transfer in systems may be required. We plan to revisit this paper and this topic at a later time to create a better model for the design of such enclosures. However, we can still present some conclusions.

## VIII. Conclusions

Though our MATLAB model does little to capture the details of our system, we can still draw some conclusions from the experimental data collected.

**VIII.1.0 Power Savings.** First, we can say that the enclosure significantly reduces the power required to maintain the build-plate temperature by almost 21% for a 70°C build-plate. Further experimentation would need to be done to determine the power savings at other temperatures.

**VIII.2.0 Plate Heating Time.** Our results and experiments did not produce any results on whether the time it takes to heat the build-plate decreases with the presence of the enclosure or not. This was not tested for and is left to future studies.

**VIII.3.0 Controlling Ambient Temperature w/ the Build-Plate.** Our experiments show that controlling the temperature inside the enclosure solely using the build-plate is *possible* however not ideal. It took the enclosure close to 30min to heat to its final temperature using just the build-plate alone as a heat source. However, after pre-heating the temperature within the enclosure was impressively maintained. We suggest the following for improved print quality from this.

- Pre-heat the enclosure for some time before starting the print.
- It may be possible to pre-heat the enclosure quicker if the glass is removed from the print-bed before pre-heating and is replaced after the temperature reaches the desired temperature.

Pre-heating the enclosure before the part starts printing can provide significant advantages to part strength. For example, many sources—such as [22]—indicate that enclosures allow parts to stay at warmer temperatures during printing so they may cool and shrink more uniformly leading to better layer adhesion and increased part strength.

**VIII.4.0 Future Work.** As mentioned in VII.2.0, future work could look into making the MATLAB model more useful so that it could be used for design. Some other ideas for modeling are presented below.

- The use of software such as Fusion360 or Solidworks as well as a CAD model to produce accurate thermal simulations for design.

Ultimately the goal is to be able to design enclosures more accurately and to analyze enclosures that currently exist. But this study overall does yield some design recommendations for printer enclosures in general.

**VIII.5.0 Design Recommendations.** We will now list the design takeaways from these experiments and model-development.

- Increasing wall thickness decreases energy loss to the surrounding environment.
- Utilizing materials with a low “k” value decreases energy loss to the surrounding environment.
- Creating an enclosure of similar size to the printer is preferable. The larger the enclosure, the less control over ambient temperature and more power loss to the environment there will be.
- Bang-bang control may not be the best way to save power in three-D printer systems. PID controllers are recommended to save more power.\*

*\*This could also have been a major contributing factor in the inaccuracy of the model. Had the build-plate been based on PID control, it is possible the model would align more with experimental results so it is important not to discount the model without looking into the effects of the control algorithm.*

We cannot comment on emissivity/reflectivity of the materials as this was not a component of the study.

**VIII.6.0 Final Thoughts.** The goal of this paper was to determine the effectiveness of a printer enclosure of our own design, hopefully develop a mathematical model to describe the enclosure, and comment on design decisions that printer enclosures may

be able to benefit from. We were able to determine the effectiveness of our enclosure, however the model created needs further evaluation to determine its usefulness and applicability to other systems. We did, however, identify some key areas on the design concepts of printer enclosures that may help design better enclosures and more efficient printers in the future. We also developed and discussed ideas for printing stronger parts using the current printer.

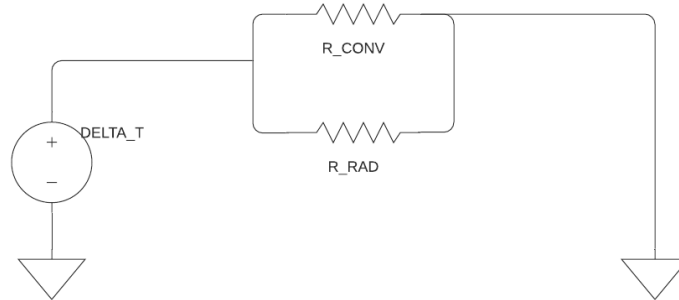
## IX. References

- [1] *Calculating the Heat Transfer in a House*: <https://dothemath.ucsd.edu/2012/11/this-thermal-house/>
- [2] *Introduction to Heat Transfer*: [https://ocw.mit.edu/courses/aeronautics-and-astronautics/16-050-thermal-energy-fall-2002/lecture-notes/10\\_part3.pdf](https://ocw.mit.edu/courses/aeronautics-and-astronautics/16-050-thermal-energy-fall-2002/lecture-notes/10_part3.pdf)
- [3] *Heat Transfer Between Surfaces of Different Areas*: <https://web.mit.edu/16.unified/www/FALL/thermodynamics/notes/node137.html>
- [4] *Power Required to Heat a Liquid or Fluid*: <https://www.vulcanic.com/en/calculations/to-calculate-heating-power/heating-volume-liquid/>
- [5] *Time Taken to Heat a Volume of Air*: <https://sciencing.com/calculate-time-heat-object-8223103.html>
- [6] *The Heat Equation*: <https://tutorial.math.lamar.edu/classes/de/theheatequation.aspx>
- [8] *Thermal Conduction Khan Academy*: <https://www.khanacademy.org/science/physics/thermodynamics/specific-heat-and-heat-transfer/a/what-is-thermal-conductivity>
- [9] *Thermal Convection*: <http://www.mhhe.com/engcs/mech/cengel/notes/ConvectionHeatTransfer.html>
- [10] *Fluid Boundary Layers*: <http://thermopedia.com/content/595/>
- [11] *Radiation as a Form of Heat Transfer*: [www.youtube.com/watch?v=otdK1x-WCsk](http://www.youtube.com/watch?v=otdK1x-WCsk)
- [12] *Radiation Formulations + Stefan Boltzmann Law*: <http://hyperphysics.phy-astr.gsu.edu/hbase/thermo/stefan.html>
- [13] *Black Body Radiation*: [https://en.wikipedia.org/wiki/Stefan-Boltzmann\\_constant](https://en.wikipedia.org/wiki/Stefan-Boltzmann_constant)
- [14] *Bang-Bang Control Concept Graph*: [https://www.researchgate.net/figure/Determination-of-a-bang-bang-control\\_fig4\\_334173391](https://www.researchgate.net/figure/Determination-of-a-bang-bang-control_fig4_334173391)
- [15] *Digital Clamp Multimeter*: [https://www.amazon.com/gp/product/B07Z398YWF/ref=ppx\\_yo\\_dt\\_b\\_asin\\_title\\_o02\\_s00?ie=UTF8&psc=1](https://www.amazon.com/gp/product/B07Z398YWF/ref=ppx_yo_dt_b_asin_title_o02_s00?ie=UTF8&psc=1)
- [16] *Wireless Digital Thermometer*: [https://www.homedepot.com/p/AcuRite-Wireless-Digital-Weather-Thermometer-00826HD/100654926?MERCH=REC-\\_PLP\\_Browse-\\_NA\\_-100654926-\\_N&](https://www.homedepot.com/p/AcuRite-Wireless-Digital-Weather-Thermometer-00826HD/100654926?MERCH=REC-_PLP_Browse-_NA_-100654926-_N&)
- [17] *Thermometer/Hygrometer*: [https://www.homedepot.com/p/AcuRite-Digital-Humidity-and-Temperature-Comfort-Monitor-00619HD/202260980?MERCH=REC-\\_searchViewed-\\_NA\\_-202260980-\\_N&](https://www.homedepot.com/p/AcuRite-Digital-Humidity-and-Temperature-Comfort-Monitor-00619HD/202260980?MERCH=REC-_searchViewed-_NA_-202260980-_N&)
- [18] *Milwaukee IR Thermometer*: <https://www.homedepot.com/p/Milwaukee-10-1-Infrared-Thermometer-2267-20H/207170357>
- [19] *Experimental Data*: <https://docs.google.com/spreadsheets/d/1iGQQArWywKGQkinwnBaXafNc4-5E-0ZXBzV1r9DVNQo/edit#gid=0>
- [20] *WiFi Smart Power Plug*: <https://www.belkin.com/us/p/P-F7C029/>
- [21] *Belkin Statistics Reporting*: <https://www.belkin.com/us/support-article?articleNum=229106#a3>
- [22] *3D Printer Enclosures*: [www.youtube.com/watch?v=79r5D9nCQfM](http://www.youtube.com/watch?v=79r5D9nCQfM)

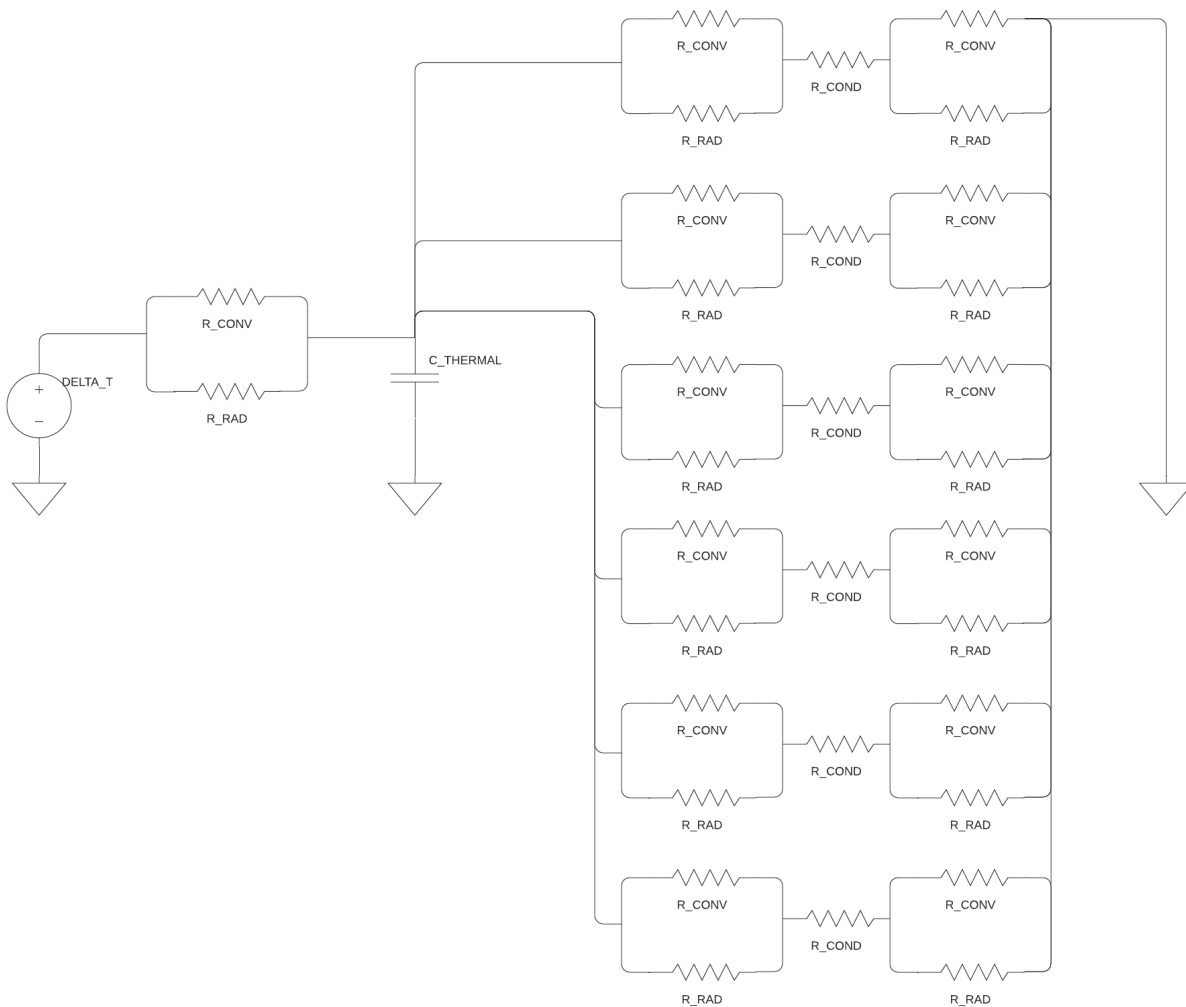


## Appendix A. Circuit Models + Schematics

### A.1.0 Equivalent Thermal Circuit Diagram of Printer w/ no Enclosure.



### A.2.0 Equivalent Thermal Circuit Diagram of Printer w/ Enclosure.



## Appendix B. MATLAB Code

```
% Environment Setup
% _____ BE SURE TO RUN THIS SECTION BEFORE ANY CALCULATION _____
clear all
close all
clc

% Define Some Initial Constants
%THERMAL CONSTANTS
sigma = 5.67*10^(-8); %W/m^2/K^4
Cp_air = 1005; %J/kg·K
d_air = 1.225; %kg/m^3 assume sea-level
k_wood = 0.12; %W/m^2/°C
k_glass = 0.8; %W/m^2/°C
%PROBLEM-SPECIFIC CONSTANTS
h = 2; %W/m^2/°C
A_plate = 0.1089; %m^2
A_glass = 0.464515; %m^2
A_wood = 0.371612; %m^2
t_glass = 0.00238125; %m
t_wood = 0.01905; %m
v_air = 0.28316847; %m^3 assume it fills case
m_air = d_air*v_air; %kg

%BOUNDARY CONDITIONS
T_plate = 70; %°C
T_room = 22; %°C, or 70°F

%NOW ONTO THE CALCULATIONS THEMSELVES

%% CALCULATION 1: Printer w/ no Enclosure

%first calculate T_REP
T_REP = (T_plate+T_room)/2+273.15; %MUST BE IN KELVIN

%calculate R_Rad + R_Conv
R_rad_plate = 1/(4*A_plate*T_REP^3*sigma);
R_conv_plate = 1/(h*A_plate);

%calculate equivalent resistance
R_TOTAL_NO_ENCLOSURE = R_rad_plate*R_conv_plate/(R_conv_plate+R_rad_plate);

%calculate the power loss in W
P_LOST_NO_ENCLOSURE = (T_plate-T_room)/R_TOTAL_NO_ENCLOSURE

%% CALCULATION 2: Printer w/ Enclosure

%__CALCULATE THERMAL RESISTANCE OF GLASS____
%NOTE: We will change T_REP later if we can
R_rad_inner_glass = 1/(4*A_glass*T_REP^3*sigma);
R_conv_inner_glass = 1/(h*A_glass);
R_rad_outer_glass = 1/(4*A_glass*T_REP^3*sigma);
R_conv_outer_glass = 1/(h*A_glass);
R_cond_glass = t_glass/(A_glass*k_glass);

%combine and simplify resistance for glass
R_INNER_GLASS = R_rad_inner_glass*R_conv_inner_glass/(R_rad_inner_glass+R_conv_inner_glass);
R_OUTER_GLASS = R_rad_outer_glass*R_conv_outer_glass/(R_conv_outer_glass+R_rad_outer_glass);
%GET R_GLASS NET
R_GLASS = R_INNER_GLASS+R_OUTER_GLASS+R_cond_glass;
```

```

%__CALCULATE THERMAL RESISTANCE OF WOOD__
%NOTE: We will change T_REP later if we can
R_rad_inner_wood = 1/(4*A_wood*T_REP^3*sigma);
R_conv_inner_wood = 1/(h*A_wood);
R_rad_outer_wood = 1/(4*A_wood*T_REP^3*sigma);
R_conv_outer_wood = 1/(h*A_wood);
R_cond_wood = t_wood/(A_wood*k_wood);

%combine and simplify resistance for wood
R_INNER_WOOD = R_rad_inner_wood*R_conv_inner_wood/(R_rad_inner_wood+R_conv_inner_wood);
R_OUTER_WOOD = R_rad_outer_wood*R_conv_outer_wood/(R_conv_outer_wood+R_rad_outer_wood);
%GET R_WOOD NET
R_WOOD = R_INNER_WOOD+R_OUTER_WOOD+R_cond_wood;

%__CALCULATE THERMAL RESISTANCE OF (4x) GLASS & (2x) WOOD__
R_TOTAL_ENCLOSURE = 1/(2/R_WOOD+4/R_GLASS)+R_TOTAL_NO_ENCLOSURE;

%CALCULATE TOTAL POWER LOSS @ STEADY STATE
P_LOST_ENCLOSURE = (T_plate-T_room)/R_TOTAL_ENCLOSURE

%CALCULATE TEMP OF AIR INSIDE
T_INSIDE = T_plate-P_LOST_ENCLOSURE*R_TOTAL_NO_ENCLOSURE

%CALC THERMAL CAPACITANCE OF AIR
C_AIR = m_air*Cp_air;
%ENERGY STORED IN THE AIR
ENERGY_STORED_AIR = C_AIR*(T_INSIDE-T_room); %assuming air starts @ room temp

%% TRANSIENT AND OTHER VALUES

%calculate power savings w/ enclosure
P_SAVED = P_LOST_ENCLOSURE-P_LOST_NO_ENCLOSURE
%temp diff inside and outside enclosure
T_DIFF = T_INSIDE-T_room

%setup the system tranfer function
s = tf('s');
H_OUT = 1/(s*C_AIR)*1/(2/R_WOOD+4/R_GLASS)/(1/(2/R_WOOD+4/R_GLASS)+1/(s*C_AIR));
H_SYS = H_OUT/(R_TOTAL_NO_ENCLOSURE+H_OUT);

%plot graph that describes temperature of the air as a function of time
opt = stepDataOptions('StepAmplitude', (T_plate-T_room));
figure;
hold on
step(H_SYS,opt);
xlabel('Time in (s)');
ylabel('Temp Increase of Air above Room Temp (°C)');
title('Temperature of Air in Enclosure vs Time');
hold off

```

### C.1.0 Experimental Procedures + Notes.

adim@mit.edu | NOT FOR PUBLICATION Page 20 of 23

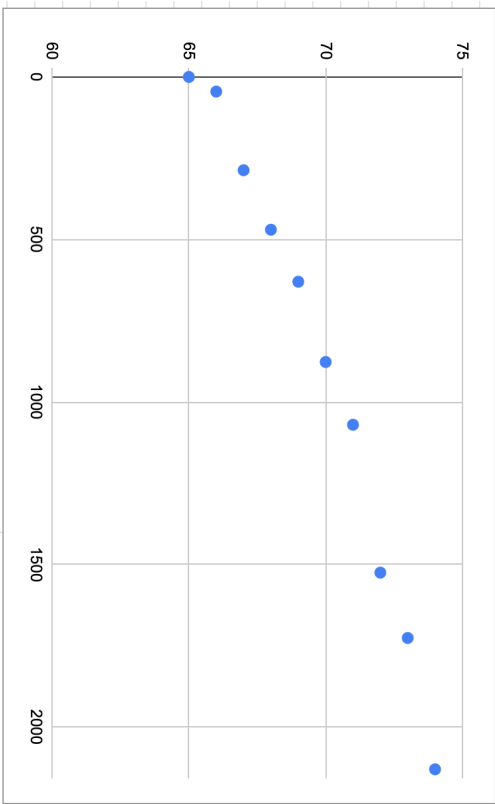


adim@mit.edu | NOT FOR PUBLICATION

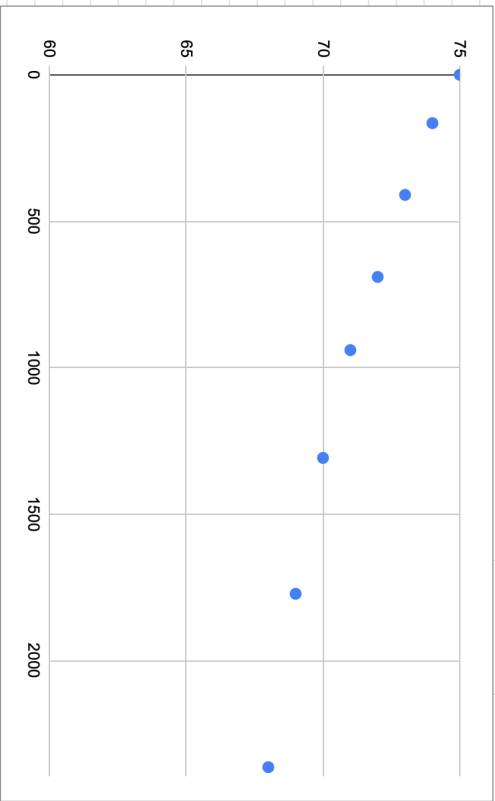
Page 21 of 23

C.3.0 Graphs + Transient Analysis.

Graph of Transient (heating):



Graph of Transient (cooling):



## Appendix D. WeMo Power Use Data

Data of interest is highlighted in RED (Data from 12/17/2020).

Exported Data for Filament Labs Caster							
Last Updated	Device Name	Device MAC	Signal Strength (%)	Energy Cost per kWh	Currency	Threshold (Watts)	
2020/12/17 15:28	Filament Labs Caster	9410E39B4C	96	0.1100 \$		8	
Daily Usage Summary							
Date	Time ON (Hours:Minutes)	Power Consumption ON (kWh)	Time STANDBY (Hours:Minutes)	Power Consumption STANDBY (kWh)	Average Day Connected Device is ON (Hours)	Average Power When ON (Watts)	Estimated Monthly Cost
2020/12/17	4:0	0.47597 0.0	0.00000	0.00000	3.55349	120	0.34770 0.05524
2020/03/24	0:0	0.00000 2:50	0.00000 2:50	0.00741	3.27264	18	0.23981 0.00082
2020/03/23	2:16	0.04566 21:43	0.00707	0.00585	3.44472	19	0.24572 0.00585
2020/03/22	0:0	0.00000 24:0	0.00772	0.00198	3.45198	19	0.24613 0.00086
2020/03/21	0:0	0.00000 24:0	0.00773	0.00773	3.82988	19	0.27164 0.00086
2020/03/20	0:0	0.00000 24:0	0.00773	4.19141	4.19141	19	0.29843 0.00086
2020/03/19	0:0	0.00000 24:0	0.00772	4.56716	4.56716	19	0.32345 0.00086
2020/03/18	14:7	0.26363 9:52	0.00223	4.56577	4.56577	19	0.32238 0.02982
2020/03/17	7:9	0.14432 16:50	0.00582	3.98839	4.56577	21	0.29131 0.01667
2020/03/16	7:49	0.16397 16:10	0.00576	3.76609	4.56577	21	0.27827 0.01884
2020/03/15	6:23	0.12807 17:36	0.00649	3.77218	3.77218	21	0.27974 0.01454
2020/03/14	8:4	0.21845 15:55	0.00569	3.31177	3.31177	13	0.24866 0.02466
2020/03/13	0:0	0.00000 24:0	0.00771	3.41385	3.41385	20	0.24727 0.00086
2020/03/12	0:0	0.00000 24:0	0.00773	4.12540	4.12540	20	0.29584 0.00086
2020/03/11	0:0	0.00000 24:0	0.00774	4.24065	4.24065	20	0.30511 0.00086
2020/03/10	2:27	0.04676 21:32	0.00697	4.40889	4.40889	20	0.31798 0.00630
2020/03/09	2:19	0.04670 21:40	0.00715	4.66220	4.66220	20	0.33662 0.00598
2020/03/08	5:17	0.10656 17:42	0.00577	4.80659	4.80659	20	0.34402 0.01247
2020/03/07	5:3	0.10198 18:56	0.00610	4.81817	4.81817	20	0.34985 0.01200
2020/03/06	5:32	0.11203 18:27	0.00615	4.98319	4.98319	21	0.36011 0.01312
2020/03/05	0:0	0.00000 24:0	0.00771	4.83399	4.83399	22	0.34865 0.00086
2020/03/04	5:53	0.13227 18:6	0.00593	5.00933	5.00933	22	0.36673 0.01534
2020/03/03	4:3	0.08201 19:56	0.00649	5.02685	5.02685	21	0.36240 0.00982
2020/03/02	7:52	0.17762 16:7	0.00531	4.81456	4.81456	23	0.34880 0.02031
2020/03/01	0:0	0.00000 24:0	0.00772	4.68889	4.68889	22	0.33882 0.00086
2020/02/29	9:18	0.20870 14:41	0.00484	5.04629	5.04629	22	0.39647 0.02370
2020/02/17	4:0	0.47987 0:0	0.00000	3.55349	3.55349	120	0.34770 0.05524
2020/02/28	9:57	0.20137 14:2	0.00463	4.36264	4.36264	21	0.31609 0.02287
2020/02/27	1:36	0.03841 22:23	0.00754	3.74256	3.74256	23	0.27957 0.00510
2020/02/26	2:24	0.05416 21:35	0.00707	3.62619	3.62619	21	0.26419 0.00680
2020/02/25	6:16	0.12686 17:43	0.00591	3.86891	3.86891	20	0.27544 0.01474
2020/02/24	4:7	0.08292 19:52	0.00567	3.59165	3.59165	21	0.26156 0.00933
2020/02/23	5:26	0.12213 18:33	0.00590	3.41054	3.41054	22	0.25902 0.01426
2020/02/22	7:22	0.14860 16:37	0.00535	3.01655	3.01655	20	0.22934 0.01709
2020/02/21	3:20	0.06709 20:39	0.00681	2.72323	2.72323	21	0.20027 0.00820
2020/02/20	3:46	0.08504 20:13	0.00610	2.69875	2.69875	22	0.20033 0.01016
2020/02/19	5:0	0.10131 18:59	0.00615	2.49407	2.49407	21	0.18624 0.01193
2020/02/18	1:2	0.02725 22:57	0.00780	2.46446	2.46446	24	0.18219 0.00389
2020/02/17	5:57	0.13381 18:2	0.00594	2.55446	2.55446	23	0.18825 0.01551
2020/02/16	5:13	0.11711 18:46	0.00590	2.22123	2.22123	23	0.16391 0.01368
2020/02/15	0:0	0.00000 24:0	0.00771	2.12871	2.12871	22	0.15556 0.00086
2020/02/14	0:49	0.02214 23:10	0.00764	2.60431	2.60431	22	0.18801 0.00331
2020/02/13	0:0	0.00000 24:0	0.00773	3.55704	3.55704	21	0.24954 0.00086
2020/02/12	5:2	0.10198 18:57	0.00622	4.33758	4.33758	21	0.30980 0.01201
2020/02/11	3:3	0.00862 20:56	0.00695	4.57438	4.57438	22	0.30839 0.01100
2020/02/10	1:45	0.03537 22:14	0.00724	4.67575	4.67575	20	0.33448 0.00473
2020/02/09	0:0	0.00000 24:0	0.00772	4.55288	4.55288	21	0.32607 0.00086